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Rank Analysis of Variance in Groups of Experiments

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Summary

An attempt has been made to suitably extend the well known Friedman's two-way analysis of variance for ranked data by using years (places) as the additional factor. The suggested method utilises none of the stringent assumptions required for the analysis of variance technique and is particularly useful for the interpretation of data from groups of experiments when the errors are heterogenous and the interaction effect is non significant. A non parametric test of the interaction effect has also been developed. The methodology has been applied for the interpretation of data obtained from a long term manurial trial.

Key words : Groups of experiments, Friedman's test, Heterogeneity chi-square, Duncan's multiple range test.

Introduction

In large scale experimental programmes it is a common practice to repeat an experiment at several places or over different localities before making valid recommendations about the suitability of treatments to varying tracts. In such case data are combined over years (or locations) to make a joint statistical analysis of the entire data by using the analysis of variance technique. But the combined analysis of data using the above method referred to as analysis of groups of experiments is not valid when the errors are heterogeneous and treatment x year interaction is absent. According to Rao [8] about 30% of heterogenous experimentation belongs to this category and as such deserves serious attention. Further, it is a well known fact that in many repetative trials observations in different plots in successive years need not necessarily be independent and the underlying distribution may depart from the usual normal law.

Hence as a safer alternative non-parametric procedures have been proposed by several workers. Rai and Rao [7] developed the K statistic from ranked data as a non-parametric alternative for the analysis of data from groups of experiments. But the method makes use of the assumption of normality of rank sums and is applicable only when the number of replications per treatment is four or more. In this paper an attempt has been made to suitably extend the well known Friedman's two-way analysis of variance for ranked data to the

case of a three way classification by taking years/locations as the additional factor. The method has been further utilised to serve as a viable alternative to the analysis of data collected from groups of experiments where the ordinary analysis of variance cannot validly be applied.

2. *Materials and Methods*

The procedure involves first ranking the observations in each block of the individual experiment. Consider a set of 't' treatments assigned randomly to r blocks of a randomised block layout. Let x_{ij} denotes the observation of treatment 'j' in block 'i' (i = 1, 2, . . . , r, j = 1, 2, . . . , t). The individual observations, x_{ij} are ranked by giving rank 1 to the highest, rank 2 to the next lower and so on, rank 't' to the lowest value. Let R_j denote the rank total of the jth treatment in the ith block. Then

$$E(R_{ij}) = \frac{t+1}{2} \qquad Cov(R_{ij}, R_{jk}) = \frac{t+1}{12}$$

$$V(R_{ij}) = \frac{t^2-1}{12}$$

where E,V and Cov stand respectively for expectation, variance and covariance. The sum of squares of deviations of the observed column totals around its expected value(s) is a measure of the differences in treatment effect.

$$S = \sum_{j=1}^{t} \left[R_j - \frac{r(t+1)}{2} \right]^2$$

under the null hypothesis of no difference between tratments the sampling distribution of S has been worked out and tables prepared by Kendall [3]. The expectation and variance of S are given by

$$E(S) = \frac{rt(t^2-1)}{12}$$

$$Var(S) = \frac{t^2 r(r-1)(t-1)(t+1)^2}{72}$$

Friedman [2] has shown that a linear function of S which is denoted as χ_r^2 is distributed approximately as a chisquare variate with $(t-1)^r$ degrees of freedom

$$\chi_r^2 = \frac{12\,s}{rt(t+1)} = \frac{12 \sum_{j=1}^{t} R_j^2}{rt(t+1)} - 3r(t+1)$$

The first two moments of χ_r^2 are (t – 1) and 2 (t – 1) which are the first two moments of a chisquare distribution with (t – 1) degrees of freedom. The higher moments of χ_r^2 are also closely approximated by corresponding higher moments of the chisquare. Thus for all practical purposes χ_r^2 can be considered to be a chisquare variable with (t – 1) degrees of freedom. Numberical comparison has shown this to be a good approximation as long as t > 7

The region of rejection for a test of equal treatment effects with level of significance, α is

$$F \in R \text{ for } f > \chi^2_{n-1,\alpha}$$

where R is the critical region and f the calculated value of F

The above approach is related to the classical analysis of variance using ranked data.

If S_T denote the total sum of squares of deviations of all the rt ranks around its average value then

$$S_T = \sum_{i=1}^{r}\sum_{j=1}^{t}\left(R_{ij} - \frac{t+1}{2}\right)^2 = \frac{rt\,(t^2-1)}{12}$$

$$\chi_r^2 = \frac{(t-1)S}{S_T}$$

The total sum of squares of the ranked data can be partitioned into two components as follows :

$$S_T = \sum_i \sum_j (R_{ij} - \bar{R}_j)^2 + \frac{S}{r} = S_R + \frac{S}{r}$$

where S_R is the residual sum of squares.

All these can be presented in an analysis of variance table as follows :

ANOVA

Source	d.f.	S.S	MSS
Between columns (treatments)	t – 1	s/r	MST
Between rows (blocks)	r – 1	0	0
Residual	(t – 1) (r – 1)	S_T – s/r	MSS
Total	tr – 1	S_T	

The additive property of chisquare enables us to extend this result to the case of three way tables with years as the additional factor. Let us assume that the set of experimental years represent a random sample from an infinite population of years. Then it is possible to calculate the Friedman's χ_r^2 statistic to the data of each of the p years separately. On the assumption of independence these chi-square values can be pooled to get a total chi-square with p (t – 1) degrees of freedom. This chisquare can be split into two components.

$$\chi_r^2 T = \chi_r^2 D + \chi_r^2 H$$

where $\chi_r^2 D$ is the deviation chi-square calculated from the column totals of the pooled data. It can be used to provide a general test of equality of treatment effects over all the p years. $\chi_r^2 H$, the heterogeneity chi-square is a component of interaction between seasons and treatments. A significant $\chi^2 H$ indicates the presence of treatment x years interaction. The relevant procedure is outlined below.

The results can also be presented in the form of an analysis of variance table as follows :

Years	S.S	Chi-square	d.f.
1	S_1	$\chi^2 r_1$	t – 1
2	S_2	$\chi^2 r_2$	t – 1
–	–	–	–
–	–	–	–
–	–	–	–
–	–	–	–
p	S_p	$\chi^2 r_p$	t–1
Total	$S_T = \sum_{i=1}^{p} S_i$	$\chi_r^2 T = \sum_{i=1}^{p} \chi_{r_i}^2$	p(t–1)
Deviation	S_D	$\chi_r^2 D = S_D \frac{12}{rp(t+1)}$	t–1
Heterogeneity	$S_H = S_T - S_D$	$\chi_r^2 H = \chi_r^2 T - \chi_r^2 D$	(p–1) (t–1)

The results can also be presented in the form of an analysis of variance table as follows :

ANOVA

Source	d.f.	*S.S*
Treatments	$t-1$	SD
Replications	$r-1$	0
Years	$p-1$	0
Treatment x year interaction	$(t-1)(p-1)$	S_H
Residual	$(r-1)(tp-1)$	S_R
Total	$rtp-1$	S_G

where,

$$S_D = \sum_{j=1}^{t} \frac{R_{.j.}^2}{rp} - \frac{rpt(t+1)^2}{4}$$

$$S_G = \sum_i \sum_j \sum_k R_{ijk}^2 - \frac{rpt(t+1)^2}{4} = rtp\frac{(t^2-1)}{12}$$

$$S_H = \sum_j \sum_k \left(\frac{R_{.jk}^2}{r}\right) - \sum_j \frac{R_{.j.}^2}{rp}$$

and $$S_R = S_G - S_D - S_H$$

Zar [9] gives a non parametric multiple comparison procedure to be adopted in two way analysis with ranks when the usual assumption of normality and homosedasticity are not satisfed. According to him rank sums are to be arranged in descending order. Critical ranges of different lengths have to be calculated by multiplying the standard error of treatment totals by the tabulated value of studentised range with number of means k and error degrees of freedom (f). Then the procedure by Newman [6] and Kaul [4] may be used for making multiple comparisons. The standard error (SE) is calculated by the expression.

$$SE(R_j) = \sqrt{\frac{rt(t+1)}{12}}$$

Among the different multiple comparison procedures multiple range test proposed by Duncan [1] is considered to be the most precise and powerful and has been widely used. Thus it would be better to incorporate a non parametric multiple comparison procedure involving Duncan's multiple range test. For the overall comparison of treatment totals based on pooled data for p years,

$$SE(R_j) = \sqrt{\frac{rtp(t+1)}{12}}$$

The critical range can be calculated from the expression,

$$w_j = D_{(n,f)} SE(R_j)$$

If treatment means are to be compared the expression becomes, $w_j = D_{(n,f)} SE(R_j)$

$$SE(R_j) = \sqrt{\frac{t(t+1)}{12rp}}$$

Here $D_{(n,f)}$ is the table value obtained from the Duncan's table with number of means n and error degrees of freedom 'f'. A range of j treatment means can be compared by w_j. In case L.S.D. is used for making multiple comparisons the relevant expression is as given below

$$L.S.D. = 1.96\sqrt{\frac{t(t+1)}{6rp}}$$

3. *Numerical Example*

As an illustration of the method described above the data relating to the permanent manurial trial on paddy for a period of 12 years from 1973 collected from the Regional Agricultural Research Station, Pattambi were utilised. The experiment was laid out in a 4 replicate randomised block design with 8 treatments. The treatments are given below.

1. Cattle manure at 18000 kg/ha to supply 90 kg N/ha
2. Green leaf at 18000 kg/ha to supply 90 kg N/ha
3. Cattle manure at 9000 kg/ha + green leaf at 9000 kg/ha to supply 90 kg N/ha
4. Ammonium sulphate to supply 90 kg N/ha
5. Cattle manure at 9000 kg/ha + Ammonium sulphate to supply 45 kg N/ha + 45 kg P_2O_5/ha + 45 kg K_2 0/ha as M.O.P.
6. Green leaf at 9000 kg/ha + Ammonium sulphate to supply 45 kgN/ha + superphosphate to supply 45 kg P_2O_5/ha + 45kg$K_2$0/ha as M.O.P.
7. Cattle manure 4500 kg/ha + Green leaf 4500 kg/ha + 45 kgN/ha as Ammonium sulphate + 45 kg P_2O_5/ha + 45kg $K_2$0/ha.
8. Ammonium sulphate to supply 45 kg N/ha + superphosphate to supply 45 kg P_2O_5/ha + M.O.P to supply 45 kg $K_2$0/ha

The observations in each block were ranked for different treatments and the sums of ranks are given in Table 1. The random variable S and the value of χ_r^2 for different years were calculated and are presented in Table 2.

Table 1. Rank sums of treatments in different years
Sums of ranks (R_j)

Years	*Treatment*								
	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	Total
1973	25	26	20	20	9	18	5	21	144
1974	17	24	14	27	4	22	12	24	144
1975	21	28	14	31	5	14	9	22	144
1976	8	25	15	25	8	24	19	20	144
1977	17	20	10	29	7	22	11	28	144
1978	10	20	13	29	6	26	13	27	144
1979	12	28	9	25	11	26	14	19	144
1980	4	26	11	26	10	27	18	22	144
1981	11	12	11	27	17	24	14	28	144
1982	6	22	12	28	8	28	17	23	144
1985	8	23	8	32	10	24	14	25	144
1987	4	26	14	31	10	20	13	26	144
Total	143	280	151	330	105	275	159	285	1728

Table 2. Values of rendom variables 'S' and χ_r^2 for different years

Years	S	χ_r^2	d.f.
1973	380	15.8333	7
1974	418	17.4167	7
1975	567	24.0000	7
1976	348	14.5000	7
1977	476	19.8333	7
1978	528	22.0000	7
1979	396	16.5000	7
1980	534	22.2500	7
1981	368	15.3333	7
1983	522	21.7500	7
1985	586	24.4167	7
1987	602	25.0833	7
Total		238.9166	84

The deviation chi-square (175.20) for the overall data was found to be statistically significant indicating that there were significant differences among the treatments in their effects. The heterogeneity chisquare (63.71) for treatmen x year interaction was not found to be statistically significant. Therefore the hypothesis that treatment effects were invariant under varying seasons was not rejected. An analysis of variance of the whole procedure mentioned above is presented in Table 3.

Table 3. Analysis of variance and chi-square values in the case of extended Friedman's test

Source	d.f.	S.S.	χ^2
Treatment	7	1051.2083	175.2017**
Year	11	0.0000	
Replication	3	0.0000	
Treatment x year interaction	77	382.2900	63.7149
Residual	285	582.5017	
Total	383	2016.0000	

** significant at 1% level

For comparative purpose the same data were analysed by the analysis of variance technique for groups of experiments. The F value for testing the significance of treatment effect was 24.4887 with 7 and 252 degrees of freedom and hence the treatment effects were found to be significant at 1% level. These observations are in general conformity with the results obtained in the non parametric approach. When multiple comparison were made among treatment means using Duncan's multiple range test the two methods produced essentially the same structure of grouping with regard to the homogenity of treatment means as is evident from the presentation given below :

1.	Groups of experiments	$T_5\ T_1\ T_3\ T_7$	$T_6\ T_2\ T_8\ T_4$
2.	Non parametric approach	$T_1\ T_5\ T_3\ T_8$	$T_8\ T_6\ T_2\ T_4$

The newly developed procedure has also been compared with the ranking mehod proposed by Rai and Rao [7]. The chi-square values for treatment and interaction components calculated in the procedure developed by Rai and Rao [7] were found to be 200.07 and 71.36 respectively. Although the chi-square values in the method proposed by Rai and Rao [7] are some what larger than that observed in the newly developed procedure the difference is very small as to deserve any serious attention. The newly developed procedure has a distinct advantage over all other methods in the sense that it is entirely distribution free, in the real sense of the term. It does not even make use of the assumption

of normality of rank sums. Rai and Rao [7] in their study have shown empirically that in majority of the cases analysed by them, the probability levels Yielded by the parametric and the ranking methods were essentially the same.

Thus, the method developed in this paper as an extention of Friedman's two way analysis of variance by ranks may be suggested as a viable alternative for the analysis of data from groups of experiments.

4. *Remark*

For small number of treatments the test proposed by Quade [5] is known to be more powerful than the Friedman's test. Hence an extension of the Quade test for groups of experiments might be better than the test proposed in this paper for extremely small number of treatments. However for large number of treatments both the tests are equally powerful. Thus the procedure presented in this paper is specially suited for the analysis and interpretaion of data from groups of experiments involving relativly larger number of treatments.

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